Microscopic mechanisms of magnetization reversal

Vladimir L. Safonov

Center for Magnetic Recording Research,
University of California -San Diego,
9500 Gilman Drive, La Jolla, CA 92093-0401

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Two principal scenarios of magnetization reversal are considered. In the first scenario all spins perform coherent motion and an excess of magnetic energy directly goes to a nonmagnetic thermal bath. A general dynamic equation is derived which includes a tensor damping term similar to the Bloch-Bloembergen form but the magnetization magnitude remains constant for any deviation from equilibrium. In the second reversal scenario, the absolute value of the averaged sample magnetization is decreased by a rapid excitation of nonlinear spin-wave resonances by uniform magnetization precession. We have developed an analytic **k**-space micromagnetic approach that describes this entire reversal process in an ultra-thin soft ferromagnetic film for up to 90° deviation from equilibrium. Conditions for the occurrence of the two scenarios are discussed.

I. INTRODUCTION

Studies of magnetization reversal in ultra-thin ferromagnetic films under an applied external magnetic field are of great importance in magnetic recording physics. A conventional theoretical tool to study magnetization reversal is based on the phenomenological Landau-Lifshitz equation [1] or, its equivalent modification with the Gilbert form of relaxation [2]. These equations conserve the absolute value of magnetization ($|\mathbf{M}| =$ const) in a single domain region. Both equations were introduced (a) for small magnetization motions and (b) for the case of uniaxial magnetic symmetry. The energy losses are defined by an isotropic phenomenological damping fitting parameter α ("damping constant").

Recently a theoretical approach [3], [4], [5], [6], [7] has been developed to correct the limitations of the Landau-Lifshitz-Gilbert (LLG) theory. The main idea was to represent the magnetization dynamics as the motion of a damped nonlinear oscillator with the random force of thermal fluctuations. The oscillator model is a convenient tool to establish a "bridge" between the microscopic physics, where the rotational oscillator complex variables naturally describe spin excitations and the macroscopic magnetization dynamics. It has been rigorously shown by including specific coupling of a magnetic system to a variety of loss mechanisms [8], [9] that for small oscillations near equilibrium the macroscopic damping term reflects the anisotropy of the system.

Our aim in this paper is to develop a self-consistent picture that describes the entire reversal process. We consider two possible scenarios: 1) the magnetization reverses uniformly, involving nonlinear dynamic damping; 2) the magnetization reverses nonuniformly, involving the excitation of nonlinear spin waves. We give an explicit criterion for this nonuniform process for an untra-thin magnetic film.

Scenario #1: the total film magnetization $|\mathbf{M}|$ is constant during reversal. All spins perform a coherent motion (the role of non-uniform spin motions is neglected). An excess of magnetic energy goes directly to a nonmagnetic thermal bath. We derive a general magnetization dynamic equation from a nonlinear oscillator model. The nonlinear damping follows from the variety of well-known physical damping mechanisms [7], [8]. Here we extend our previous results for uniaxial symmetry to the case of non-uniaxial symmetry.

Scenario #2: the total film magnetization $|\mathbf{M}|$ decreases. Experimentally it was observed in Ref. [10]. Large angle magnetization motion can excite spin-wave instabilities, which increase substantially the magnetization reversal rate [11]. We explicitly evaluate the second order Suhl instability and construct a self-consistent theory of magnetization switching for up to 90° from equilibrium.

The problem of nonlinear spin-wave excitation during reversal has been explored by numerical simulations in nanograins [12], [13], and thin films [14], [15]. All these simulations have been performed using conventional local micromagnetic modeling, which includes: a) the analysis of intra- and inter-cell interactions, b) analysis of phenomenological dynamic equations, and c) computer simulations. There are two principal problems in this technique: 1) the physical problem of the introduction of local phenomenological damping (and corresponding magnetic noise) and 2) the computing problem in the case of a large number of cells.

Both problems of local micromagnetic modeling can be avoided by developing the **k**-space micromagnetic modeling as we do in this work. Our theory includes: a) an analysis of spin-wave spectra and interactions in an ultra-thin film, b) the calculation of the effective scattering processes (most of the accumulated energy is to be transformed to nonlinear spin waves), c) the analysis of self-consistent dynamic equations with microscopic damping (and noise, if necessary). Note that a similar technique to study nonlinear spin-wave dynamics has been developed

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in the theory of parametric magnon excitation (mainly in the bulk, see, e.g., [16], [17], [18], [19]). We have already considered **k**-space modeling in application to magnetic noise in a thin film [20]. Recently Dobin and Victora [21] estimated the increment of the second order Suhl instability and corresponding effective magnetization reversal time for up to 25° deviation from equilibrium in the film plane. Here we give an explicit analytic formulation to describe magnetization reversal (switching) for up to 90° deviation from equilibrium in ultra-thin films in terms of spin-wave pair excitations.

II. SCENARIO #1: $|\mathbf{M}| = $ CONST

In this section we consider the magnetization reversal without spin-wave excitations. The approach is to transform the magnetization dynamics without damping to normal mode coordinates. Then we introduce nonlinear damping, which has a connection with microscopic physics and return back to magnetization coordinates. The analysis parallels the approach for low-level excitations.

Let us consider small-amplitude magnetization motions of a single-domain ferromagnetic particle in the vicinity of equilibrium state $\mathbf{M}||\widehat{\mathbf{z}}_0$, where $\widehat{\mathbf{z}}_0$ is the unit vector in the equilibrium direction. The magnetization rotation around effective field in this case, in general, is elliptical and the magnetic energy $\mathcal{E}$ can be represented as a quadratic form:

$$\mathcal{E}/\left(\frac{M_s V}{\gamma}\right) = \frac{\gamma H_{x_0}}{2}m_{x_0}^2 + \frac{\gamma H_{y_0}}{2}m_{y_0}^2. \quad (1)$$

Here $\mathbf{m} \equiv \mathbf{M}/M_s$, $\widehat{\mathbf{x}}_0$ and $\widehat{\mathbf{y}}_0$ are the unit orthogonal vectors in the plane perpendicular to the equilibrium direction, M_s is the saturation magnetization and V is the particle volume. H_{x_0} and H_{y_0} are positive Kittel "stiffness" fields, which include both microscopic and shape anisotropies and the external magnetic field. The parameter $M_s V/\gamma \equiv \hbar S$, where $\hbar$ is Planck's constant and S is the total spin of the film.

From the Holstein-Primakoff transformation [22] we have:

$$m^+ = a\sqrt{1+m_{z_0}}, \quad m_{z_0} = 1 - a^* a, \quad (2)$$
$$m^- = a^*\sqrt{1+m_{z_0}}, \quad m^{\pm} = m_{x_0} \pm i m_{y_0},$$

where a^* and a describe spin excitations.

The magnetic energy (1) can be written in the quadratic form:

$$\mathcal{E}/\left(\frac{M_s V}{\gamma}\right) = \mathcal{A}a^* a + \frac{\mathcal{B}}{2}(aa + a^* a^*), \quad (3)$$

where $\mathcal{A} = \gamma(H_{x_0}+H_{y_0})/2$ and $\mathcal{B} = \gamma(H_{x_0}-H_{y_0})/2$. The non-diagonal terms in (3) are eliminated by the linear canonical transformation (e.g., [23]):

$$a = uc + vc^*, \quad a^* = uc^* + vc, \quad (4)$$
$$u = \sqrt{\frac{\mathcal{A}+\omega_0}{2\omega_0}}, \quad v = -\frac{\mathcal{B}}{|\mathcal{B}|}\sqrt{\frac{\mathcal{A}-\omega_0}{2\omega_0}}.$$

The energy in terms of the normal mode coordinates c and c^* is simply:

$$\mathcal{E}/\left(\frac{M_s V}{\gamma}\right) = \omega_0 c^* c, \quad (5)$$

where $\omega_0 = \sqrt{\mathcal{A}^2 - \mathcal{B}^2} = \gamma\sqrt{H_{x_0}H_{y_0}}$ is the ferromagnetic resonance frequency.

The dynamic equations for c and c^* are independent and can be written as:

$$\frac{dc}{dt} + \eta c = -i\omega_0 c, \quad \frac{dc^*}{dt} + \eta c^* = i\omega_0 c^*. \quad (6)$$

Here η is the linear relaxation rate, which can be found microscopically [8]. In the case of large magnetization motion we can write a corresponding nonlinear oscillator equation in the form:

$$\frac{dc}{dt} + \eta(N)c = G(c, c^*), \quad N \equiv c^* c. \quad (7)$$

Here $G(c, c^*)$ corresponds to the gyromagnetic term $-\gamma\mathbf{m} \times \mathbf{H}_{\text{eff}}$. The nonlinear relaxation rate $\eta(N)$ can be estimated from the known relaxation process for the uniform precession [7]. We assume that $|\mathbf{m}| = 1$ and therefore no spin waves are excited.

Using back transformations (4) and (2), we can derive an equation (corresponding to (7)) in terms of **m**-components:

$$\frac{d\mathbf{m}}{dt} = -\gamma\mathbf{m} \times \mathbf{H}_{\text{eff}} - \overleftrightarrow{\Gamma} \cdot (\mathbf{m} - \widehat{\mathbf{z}}_0), \quad (8)$$

$$\overleftrightarrow{\Gamma} = 2\eta(N)\begin{pmatrix} \frac{m_{z_0}}{1+m_{z_0}} & 0 & 0 \\ 0 & \frac{m_{z_0}}{1+m_{z_0}} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where

$$N = \frac{\mathcal{A}}{\omega_0}(1-m_{z_0}) + \frac{\mathcal{B}}{\omega_0}\frac{m_{x_0}^2 - m_{y_0}^2}{1+m_{z_0}}. \quad (9)$$

Note that the Eq.(8) conserves the magnitude of **m**. For small deviations from equilibrium, when $m_{z_0} \simeq 1$, this equations exactly correspond to Bloch-Bloembergen equations [24] with $\eta(0) = 1/T_2$ and $1/T_1 = 2/T_2$. From Eq.(8) we see that one can expect the anomalously large damping in the case of 180° reversal when $1 + m_{z_0} \to 0$ (see, also [7], [25]). Simple numerical analysis shows that magnetization dynamics in the framework of Eq.(8) for angles about 70° (and smaller) can be approximated by LLG dynamics. This can explain why the switching experiment [10] with $|\mathbf{M}| = $ const was well fitted by LLG equation.

III. SCENARIO #2: $|\mathbf{M}| \neq$ CONST

We shall consider an ultra-thin ferromagnetic film ($\tau \times L_y \times L_z$) with the magnetization: $\mathbf{M}(\mathbf{r}) = M_s \mathbf{m}(\mathbf{r})$, $\mathbf{r} = (y, z)$. The variation of the unit vector $\mathbf{m}$ within the film thickness ($-\tau/2 \leq x \leq \tau/2$) will be neglected. Locally one has: $|\mathbf{m}(\mathbf{r})| = \sqrt{m_x^2 + m_y^2 + m_z^2} = 1$.

In order to introduce collective magnetization motions, we assume that the film is periodic along both y and z directions with periods L_y and L_z, respectively. The Fourier series representation can be written as

$$\mathbf{m}(\mathbf{r}) = \sum_{\mathbf{k}} \mathbf{m_k} \exp(i\mathbf{k} \cdot \mathbf{r}), \tag{10}$$

$$\mathbf{m_k} = \frac{1}{L_y L_z} \int_0^{L_y} dy \int_0^{L_z} dz \, \mathbf{m}(\mathbf{r}) \exp(-i\mathbf{k} \cdot \mathbf{r}),$$

$k_y = 2\pi n_y / L_y$, $k_z = 2\pi n_z / L_z$ ($-\infty < n_y, n_z < \infty$) are the wave vector components in the plane.

The equilibrium is supposed to be a uniformly magnetized state, in which the magnetization is oriented in the (y, z) plane along an equilibrium axis z_0. The transformation from the (x, y, z) coordinates to equilibrium coordinates (Fig.1) (x_0, y_0, z_0) is defined by

$$\begin{pmatrix} y \\ z \end{pmatrix} = \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin\theta_0 & \cos\theta_0 \end{pmatrix} \begin{pmatrix} y_0 \\ z_0 \end{pmatrix} \tag{11}$$

Here θ_0 determines a rotation in the film plane, $x = x_0$. Analogous transformation should be used for $(m_y, m_z) \rightarrow (m_{y_0}, m_{z_0})$ and wave vector components $(k_y, k_z) \rightarrow (k_{y_0}, k_{z_0})$. Note that both the absolute value of the wave vector $k = |\mathbf{k}|$ and the scalar product $\mathbf{k} \cdot \mathbf{r}$ are invariant in respect to choice of system of coordinates.

A. Magnetic energy

The magnetic energy of the film contains the exchange energy, energy of anisotropy, Zeeman energy and demagnetization energy:

$$\mathcal{E} = \mathcal{E}_{exch} + \mathcal{E}_{anis} + \mathcal{E}_Z + \mathcal{E}_{dmag}. \tag{12}$$

The exchange energy $-A(\triangledown \cdot \mathbf{m})^2$ can be represented as

$$\mathcal{E}_{exch} = VA \sum_{\mathbf{k}} k^2 \left(m_{y_0,\mathbf{k}} m_{y_0,-\mathbf{k}} + m_{z_0,\mathbf{k}} m_{z_0,-\mathbf{k}} \right), \tag{13}$$

where A is the exchange constant and $V = \tau L_y L_z$ is the film volume. To obtain (13) we have used the following formula:

$$\frac{1}{L_y L_z} \int_0^{L_y} dy \int_0^{L_z} dz \, \exp[i(\mathbf{k} + \mathbf{k}_1) \cdot \mathbf{r}] = \Delta(\mathbf{k} + \mathbf{k}_1), \tag{14}$$

where $\Delta(\cdot)$ is the Kronecker delta function: $\Delta(\mathbf{q}) = 1$, if $\mathbf{q} = 0$ and $\Delta(\mathbf{q}) = 0$ otherwise.

The quadratic uniaxial anisotropy energy (z is an easy axis, see, Fig. 1) in the $\mathbf{k}$-space is:

$$\mathcal{E}_{anis} = -VK_1 \sum_{\mathbf{k}} \left(-m_{y_0,\mathbf{k}} \sin\theta_0 + m_{z_0,\mathbf{k}} \cos\theta_0 \right)$$
$$\times \left(-m_{y_0,-\mathbf{k}} \sin\theta_0 + m_{z_0,-\mathbf{k}} \cos\theta_0 \right). \tag{15}$$

The Zeeman energy in the external magnetic field $\mathbf{H}_0 = (0, \; H_0 \sin\theta_H, \; H_0 \cos\theta_H)$ is:

$$\mathcal{E}_Z = -VM_s H_0 [m_{y_0,0} \sin(\theta_H - \theta_0) + m_{z_0,0} \cos(\theta_H - \theta_0)]. \tag{16}$$

The demagnetization energy (see, [26]) is defined by :

$$\mathcal{E}_{dmag} = 2\pi M_s^2 V \sum_{\mathbf{k}} \{ G(k\tau) m_{x_0,\mathbf{k}} m_{x_0,-\mathbf{k}} \tag{17}$$

$$+ [1 - G(k\tau)] \left[\left(\frac{k_{y_0}}{k} \right)^2 m_{y_0,\mathbf{k}} m_{y_0,-\mathbf{k}} \right.$$

$$+ \left(\frac{k_{z_0}}{k} \right)^2 m_{z_0,\mathbf{k}} m_{z_0,-\mathbf{k}} + 2 \frac{k_{y_0} k_{z_0}}{k^2} m_{y_0,\mathbf{k}} m_{z_0,-\mathbf{k}}] \},$$

where $G(x) = [1 - \exp(-x)]/x$.

B. Spin waves

We shall utilize a classical form of the spin representation in terms of Bose operators introduced in Refs. [27], [28] and convenient for 2D systems. For the unit magnetization vector $\mathbf{m}$ this representation in terms of complex variables a and a^* can be written as:

$$m_{x_0} = i\frac{a - a^*}{\sqrt{2}}, \tag{18a}$$

$$m_{y_0} = \sqrt{1 - m_{x_0}^2} \sin\left(\frac{a + a^*}{\sqrt{2}} \right), \tag{18b}$$

$$m_{z_0} = \sqrt{1 - m_{x_0}^2} \cos\left(\frac{a + a^*}{\sqrt{2}} \right). \tag{18c}$$

An expansion of (18a)-(18c) up to the fourth order gives accuracy $\sim 6\%$ for about 90° deviation from equilibrium:

$$m_{x_0} = i\frac{a - a^*}{\sqrt{2}}, \tag{19a}$$

$$m_{y_0} \simeq \frac{a + a^*}{\sqrt{2}} + \frac{a^3 + (a^*)^3 - 3a^* a^2 - 3(a^*)^2 a}{6\sqrt{2}}, \tag{19b}$$

$$m_{z_0} \simeq 1 - a^* a - \frac{a^4 + (a^*)^4 - 2a^* a^3 - 2(a^*)^3 a}{12}. \tag{19c}$$

The following Fourier representation for $a(\mathbf{r})$ (and its complex conjugate) will be used:

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$$a(\mathbf{r}) = \sum_k a_{\mathbf{k}} \exp(i\mathbf{k}\mathbf{r}_j), \qquad (20)$$

$$a_{\mathbf{k}} = \frac{1}{L_y L_z} \int\limits_0^{L_y} dy \int\limits_0^{L_z} dz \, a(\mathbf{r}) \exp(-i\mathbf{k}\cdot\mathbf{r}).$$

In general, the magnetic energy can be expressed as

$$\mathcal{E} = \mathcal{E}^{(0)} + \mathcal{E}^{(1)} + \mathcal{E}^{(2)} + \mathcal{E}^{(3)} + \mathcal{E}^{(4)} + ..., \qquad (21)$$

where the superscript denotes an order in terms of a and a^*.

The zeroth order energy term is equal to

$$\mathcal{E}^{(0)} = -V K_1 \cos^2\theta_0 - V M_s H_0 \cos(\theta_H - \theta_0). \qquad (22)$$

The equilibrium uniformly magnetized state is defined by the condition: $\partial\mathcal{E}^{(0)}/\partial\theta_0 = 0$, which corresponds to

$$H_K \sin 2\theta_0 = 2H_0 \sin(\theta_H - \theta_0). \qquad (23)$$

Here $H_K = 2K_1/M_s$ is the anisotropy field. In order to have a stable stationary state, we need $\partial^2\mathcal{E}^{(0)}/\partial\theta_0^2 > 0$. The first order energy term $\mathcal{E}^{(1)} = 0$ at the equilibrium and this condition coincides with Eq.(23).

The quadratic term has the form:

$$\mathcal{E}^{(2)}/\left(\frac{M_s V}{\gamma}\right) = \sum_k \left[\mathcal{A}_{\mathbf{k}} a_{\mathbf{k}}^* a_{\mathbf{k}} + \frac{\mathcal{B}_{\mathbf{k}}}{2}(a_{\mathbf{k}} a_{-\mathbf{k}} + a_{\mathbf{k}}^* a_{-\mathbf{k}}^*) \right], \qquad (24)$$

where

$$\mathcal{A}_{\mathbf{k}} = \gamma\alpha_E k^2 - \frac{\gamma H_K}{2}\sin^2\theta_0 + \gamma H_0 \cos(\theta_H - \theta_0)$$
$$+ 2\pi\gamma M_s \left[[1 - G(k\tau)]\left(\frac{k_{y_0}}{k}\right)^2 + G(k\tau) \right], \qquad (25)$$

$$\mathcal{B}_{\mathbf{k}} = \gamma\alpha_E k^2 - \frac{\gamma H_K}{2}\sin^2\theta_0$$
$$+ 2\pi\gamma M_s \left[[1 - G(k\tau)]\left(\frac{k_{y_0}}{k}\right)^2 - G(k\tau) \right], \qquad (26)$$

and $\alpha_E \equiv A/M_s$. Using the following linear canonical transformation (e.g., [23]):

$$a_{\mathbf{k}} = u_{\mathbf{k}} c_{\mathbf{k}} + v_{\mathbf{k}} c_{-\mathbf{k}}^*, \quad a_{\mathbf{k}}^* = u_{\mathbf{k}} c_{\mathbf{k}}^* + v_{\mathbf{k}} c_{-\mathbf{k}}, \qquad (27)$$

$$u_{\mathbf{k}} = \sqrt{\frac{\mathcal{A}_{\mathbf{k}} + \omega_{\mathbf{k}}}{2\omega_{\mathbf{k}}}}, \quad v_{\mathbf{k}} = -\frac{\mathcal{B}_{\mathbf{k}}}{|\mathcal{B}_{\mathbf{k}}|}\sqrt{\frac{\mathcal{A}_{\mathbf{k}} - \omega_{\mathbf{k}}}{2\omega_{\mathbf{k}}}}, \qquad (28)$$

we obtain

$$\mathcal{E}^{(2)} = \frac{M_s V}{\gamma} \sum_k \omega_{\mathbf{k}} c_{\mathbf{k}}^* c_{\mathbf{k}}, \quad \omega_{\mathbf{k}} = \sqrt{\mathcal{A}_{\mathbf{k}}^2 - \mathcal{B}_{\mathbf{k}}^2}. \qquad (29)$$

The spin-wave frequency, $\omega_{\mathbf{k}}$, in an explicit form is

$$\omega_{\mathbf{k}} = \gamma \Big[H_0 \cos(\theta_H - \theta_0) - H_K \sin^2\theta_0$$
$$+ 4\pi M_s[1 - G(k\tau)]\left(\frac{k_{y_0}}{k}\right)^2 + 2\alpha_E k^2 \Big]^{1/2}$$
$$\times [H_0 \cos(\theta_H - \theta_0) + 4\pi M_s G(k\tau)]^{1/2}. \qquad (30)$$

C. Spin-wave interactions

The interaction energy can be represented in the form:

$$\frac{\mathcal{E}_{int}}{(M_s V/\gamma)} = \sum_{\mathbf{1,2,3}} \Big[\frac{\Psi_1(\mathbf{1,2,3})}{3} c_1 c_2 c_3 \qquad (31)$$
$$+ \Psi_2(\mathbf{1,2;-3}) c_1 c_2 c_{-3}^* + \text{c.c.} \Big] \Delta(\mathbf{1+2+3})$$
$$+ \frac{1}{2} \sum_{\mathbf{1,2,3,4}} \Phi(\mathbf{1,2,3,4}) c_1^* c_2^* c_3 c_4 \Delta(\mathbf{1+2-3-4}).$$

Here for simplicity we use the following notations: $\mathbf{k}_1 \equiv \mathbf{1}$, $\mathbf{k}_2 \equiv \mathbf{2}$, etc., for example, $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 \equiv \mathbf{1+2+3}$. The three-wave interaction amplitudes are

$$\Psi_1(\mathbf{1,2,3}) = \frac{1}{2}\{\psi(\mathbf{1})(u_1 + v_1)(u_2 v_3 + u_3 v_2)$$
$$+ \psi(\mathbf{2})(v_1 u_3 + v_3 u_1)(u_2 + v_2)$$
$$+ \psi(\mathbf{3})(v_1 u_2 + v_2 u_1)(u_3 + v_3)\}. \qquad (32)$$

and

$$\Psi_2(\mathbf{1,2;3}) = \frac{1}{2}\{\psi(\mathbf{1})(u_1 + v_1)(u_2 u_3 + v_2 v_3)$$
$$+ \psi(\mathbf{2})(u_2 + v_2)(u_1 u_3 + v_1 v_3)$$
$$+ \psi(\mathbf{3})(u_3 + v_3)(v_1 u_2 + v_2 u_1)\}. \qquad (33)$$

Here

$$\psi(\mathbf{k}) = -\frac{\gamma}{\sqrt{2}}\left(\frac{H_K}{2}\sin 2\theta_0 + 4\pi M_s[1 - G(k\tau)]\frac{k_{y_0} k_{z_0}}{k^2} \right). \qquad (34)$$

The four-wave interaction amplitude can be expressed as:

$$\Phi(\mathbf{1,2,3,4}) = \Phi_0(\mathbf{1,2,3,4}) + \Phi_s(\mathbf{1,2,3,4}) + \Phi_Q(\mathbf{1,2,3,4}), \qquad (35)$$

where

$$\Phi_0 = [\gamma H_0 \cos(\theta_H - \theta_0) + \gamma H_K \cos^2\theta_0]$$
$$\times \Big\{ u_1 u_2 v_3 v_4 + v_1 v_2 u_3 u_4$$
$$- \frac{1}{2}[(u_1 u_2 + v_1 v_2)(u_3 v_4 + v_3 u_4)$$
$$+ (u_1 v_2 + v_1 u_2)(u_3 u_4 + v_3 v_4)] \Big\}, \qquad (36)$$

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$$\Phi_s = \frac{1}{4}\left[\mathcal{P}(\mathbf{1}) + \mathcal{P}(\mathbf{2}) + \mathcal{P}(\mathbf{3}) + \mathcal{P}(\mathbf{4})\right]$$
$$\times[(u_1 u_2 - v_1 v_2)(v_3 v_4 - u_3 u_4)$$
$$- (u_1 v_2 + v_1 u_2)(v_3 v_4 + u_3 u_4)$$
$$- (u_1 u_2 + v_1 v_2)(v_3 u_4 + u_3 v_4)$$
$$- \frac{1}{2}(v_1 u_2 + u_1 v_2)(v_3 u_4 - u_3 v_4)], \qquad (37)$$

$$\Phi_Q = [\mathcal{Q}(\mathbf{1} + \mathbf{2}) + \mathcal{Q}(\mathbf{3} + \mathbf{4})](u_1 u_2 u_3 u_4 + v_1 v_2 v_3 v_4)$$
$$+ [\mathcal{Q}(\mathbf{1} + \mathbf{4}) + \mathcal{Q}(\mathbf{2} + \mathbf{3})](u_1 v_2 u_3 v_4 + v_1 u_2 v_3 u_4)$$
$$+ [\mathcal{Q}(\mathbf{1} + \mathbf{3}) + \mathcal{Q}(\mathbf{2} + \mathbf{4})](u_1 v_2 v_3 u_4 + v_1 u_2 u_3 v_4), \quad (38)$$

$$\frac{\mathcal{P}(\mathbf{k})}{\gamma} \equiv \alpha_E k^2 - \frac{H_K}{2}\sin^2\theta_0 + 2\pi M_s[1 - G(k\tau)]\left(\frac{k_{y_0}}{k}\right)^2,$$
$$(39)$$

and

$$\frac{\mathcal{Q}(\mathbf{k})}{\gamma} \equiv \alpha_E k^2 - \frac{H_K}{2}\cos^2\theta_0 + 2\pi M_s[1 - G(k\tau)]\left(\frac{k_{z_0}}{k}\right)^2.$$
$$(40)$$

D. Effective four-wave interactions

Unitary transformation (see, [29]) makes it possible to eliminate forbidden three-magnon interaction terms in Eq.(31) and obtain effective interaction terms. As a result we have the following spin-wave energy

$$\mathcal{E}/\left(\frac{M_s V}{\gamma}\right) = \sum_{\mathbf{k}} \omega_{\mathbf{k}} c_{\mathbf{k}}^* c_{\mathbf{k}} \qquad (41)$$
$$+ \frac{1}{2}\sum_{\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}} \widetilde{\Phi}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}) c_1^* c_2^* c_3 c_4 \Delta(\mathbf{1} + \mathbf{2} - \mathbf{3} - \mathbf{4}),$$

where

$$\widetilde{\Phi}(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}) = \Phi(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}) + \Phi_1(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}) + \Phi_2(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}),$$
$$(42)$$

$$\Phi_1(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}) = -\Psi_1(\mathbf{1},\mathbf{2},-\mathbf{1}-\mathbf{2})\Psi_1(\mathbf{3},\mathbf{4},-\mathbf{3}-\mathbf{4})$$
$$\times\left(\frac{1}{\omega_1 + \omega_2 + \omega_{-1-2}} + \frac{1}{\omega_3 + \omega_4 + \omega_{-3-4}}\right), \quad (43)$$

and

$$\Phi_2(\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4}) = \Psi_2(\mathbf{1},\mathbf{2},\mathbf{1}+\mathbf{2})\Psi_2(\mathbf{3},\mathbf{4},\mathbf{3}+\mathbf{4})$$
$$\times\left(\frac{1}{\omega_1 + \omega_2 - \omega_{1+2}} + \frac{1}{\omega_3 + \omega_4 - \omega_{3+4}}\right)$$
$$- \Psi_2(\mathbf{1},\mathbf{3}-\mathbf{1},\mathbf{3})\Psi_2(\mathbf{4},\mathbf{2}-\mathbf{4},\mathbf{2})$$

$$\times\left(\frac{1}{\omega_1 + \omega_{3-1} - \omega_3} + \frac{1}{\omega_4 + \omega_{2-4} - \omega_2}\right)$$
$$- \Psi_2(\mathbf{1},\mathbf{4}-\mathbf{1},\mathbf{4})\Psi_2(\mathbf{3},\mathbf{2}-\mathbf{3},\mathbf{2})$$
$$\times\left(\frac{1}{\omega_1 + \omega_{4-1} - \omega_4} + \frac{1}{\omega_3 + \omega_{2-3} - \omega_2}\right)$$
$$- \Psi_2(\mathbf{2},\mathbf{3}-\mathbf{2},\mathbf{3})\Psi_2(\mathbf{4},\mathbf{1}-\mathbf{4},\mathbf{1})$$
$$\times\left(\frac{1}{\omega_2 + \omega_{3-2} - \omega_3} + \frac{1}{\omega_4 + \omega_{1-4} - \omega_1}\right)$$
$$- \Psi_2(\mathbf{2},\mathbf{4}-\mathbf{2},\mathbf{4})\Psi_2(\mathbf{3},\mathbf{1}-\mathbf{3},\mathbf{1})$$
$$\times\left(\frac{1}{\omega_2 + \omega_{4-2} - \omega_4} + \frac{1}{\omega_3 + \omega_{1-3} - \omega_1}\right). \quad (44)$$

The energy (41) together with the Hamilton's equations of motion for complex spin-wave variables

$$\left(\frac{d}{dt} + \eta_{\mathbf{k}}\right) c_{\mathbf{k}} = -i\left(\frac{\gamma}{M_s V}\right)\frac{\partial\mathcal{E}}{\partial c_{\mathbf{k}}^*}, \qquad (45)$$

supplemented by the (microscopic) relaxation rate $\eta_{\mathbf{k}}$, represent the basis for magnetization dynamics modeling in the $\mathbf{k}$-space. Calculating $c_{\mathbf{k}}(t)$, $c_{\mathbf{k}}^*(t)$, we can find magnetization deviations $a_{\mathbf{k}}(t)$, $a_{\mathbf{k}}^*(t)$ with the help of back transformation (27) and finally with Eqs. (19a)-(19c), the averaged

$$\langle\mathbf{m}(\mathbf{r},t)\rangle = \frac{1}{L_y L_z}\int\limits_0^{L_y} dy \int\limits_0^{L_z} dz\ \mathbf{m}(\mathbf{r},t) = \mathbf{m}_0(t), \quad (46)$$

which gives a measure of non-uniform magnetization motions in the system. In general, $|\mathbf{m}_0| < 1$ and only in the case of coherent spin motion $|\mathbf{m}_0| = 1$.

E. Example

For simplicity we shall consider the uniform magnetization precession interacting with one spin-wave pair with $\mathbf{k} = (0, 0, k)$ along $\widehat{\mathbf{z}}_0$. In this case the energy (Hamiltonian) of the system can be reduced to the form:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int} + \mathcal{H}_p, \qquad (47)$$

where

$$\mathcal{H}_0/\left(\frac{M_s V}{\gamma}\right) = \omega_0 c_0^* c_0 + \omega_{\mathbf{k}}(c_{\mathbf{k}}^* c_{\mathbf{k}} + c_{-\mathbf{k}}^* c_{-\mathbf{k}}) \qquad (48)$$

describes the uniform precession and spin-wave pair,

$$\frac{\mathcal{H}_{int}}{(M_s V/\gamma)} = \frac{\Phi_{00}}{2} c_0^* c_0^* c_0 c_0 + 2\Phi_{0\mathbf{k}} c_0^* c_0(c_{\mathbf{k}}^* c_{\mathbf{k}} + c_{-\mathbf{k}}^* c_{-\mathbf{k}}) \quad (49)$$
$$+ \frac{\Phi_{\mathbf{kk}}}{2}(c_{\mathbf{k}}^* c_{\mathbf{k}} c_{\mathbf{k}}^* c_{\mathbf{k}} + c_{-\mathbf{k}}^* c_{-\mathbf{k}} c_{-\mathbf{k}}^* c_{-\mathbf{k}}) + 2\Phi_{\mathbf{k},-\mathbf{k}} c_{\mathbf{k}}^* c_{\mathbf{k}} c_{-\mathbf{k}}^* c_{-\mathbf{k}}$$

describes nonlinear interactions in the system, $\Phi_{00} \equiv \widetilde{\Phi}(0,0,0,0)$, $\Phi_{0\mathbf{k}} \equiv \widetilde{\Phi}(\mathbf{k},0,\mathbf{k},0)$, $\Phi_{\mathbf{kk}} \equiv \widetilde{\Phi}(\mathbf{k},\mathbf{k},\mathbf{k},\mathbf{k})$, $\Phi_{\mathbf{k},-\mathbf{k}} \equiv \widetilde{\Phi}(\mathbf{k},-\mathbf{k},\mathbf{k},-\mathbf{k})$, and

$$\mathcal{H}_p / \left(\frac{M_s V}{\gamma} \right) = \Phi_p \left(c_0 c_0 c_{\mathbf{k}}^* c_{-\mathbf{k}}^* + c_0^* c_0^* c_{\mathbf{k}} c_{-\mathbf{k}} \right) \qquad (50)$$

describes the spin-wave pair excitation by the uniform precession, $\Phi_p \equiv \widetilde{\Phi}(\mathbf{k}, -\mathbf{k}, 0, 0)$.

The uniform precession and spin-wave pair dynamics are defined by

$$\left(\frac{d}{dt} + \eta_0 \right) c_0 = -i\widetilde{\omega}_0 c_0 - 2i\Phi_p c_{\mathbf{k}} c_{-\mathbf{k}} c_0^*, \qquad (51a)$$

$$\left(\frac{d}{dt} + \eta_{\mathbf{k}} \right) c_{\mathbf{k}} = -i\widetilde{\omega}_{\mathbf{k}} c_{\mathbf{k}} - i\Phi_p (c_0)^2 c_{-\mathbf{k}}^*, \qquad (51b)$$

where

$$\widetilde{\omega}_0 = \omega_0 + \widetilde{\Phi}_{00}|c_0|^2 + 2\Phi_{0\mathbf{k}}(|c_{\mathbf{k}}|^2 + |c_{-\mathbf{k}}|^2), \qquad (52)$$

$$\widetilde{\omega}_{\mathbf{k}} = \omega_{\mathbf{k}} + 2\widetilde{\Phi}_{0\mathbf{k}}|c_0|^2 + \Phi_{\mathbf{k}\mathbf{k}}|c_{\mathbf{k}}|^2 + 2\Phi_{\mathbf{k},-\mathbf{k}}|c_{-\mathbf{k}}|^2 \qquad (53)$$

are nonlinear frequencies and η_0, $\eta_{\mathbf{k}}$ are the relaxation rates.

From the energy symmetry we have $c_{\mathbf{k}} = c_{-\mathbf{k}}$ (see, also [19]). Thus, Eqs.(51a)-(53) represent a self-consistent nonlinear theory of magnetization reversal with just two independent complex variables. Simple analysis for both $c_{\mathbf{k}}(t)$ and $c_{-\mathbf{k}}^*(t) \propto \exp(\kappa t)$, where κ is an increment of instability, gives:

$$\kappa = -\eta_{\mathbf{k}} + \sqrt{\left|\Phi_p c_0^2(0)\right|^2 + (\widetilde{\omega}_{\mathbf{k}} - \widetilde{\omega}_0)^2}. \qquad (54)$$

This formula is similar to that obtained in Ref. [30] for parametric instabilities. The difference is that the resonance condition includes nonlinear frequencies: $2\widetilde{\omega}_0 = \widetilde{\omega}_{\mathbf{k}} + \widetilde{\omega}_{-\mathbf{k}}$ (all spin waves out of this equality can be included into a thermal bath) and $\widetilde{\omega}_0$ is the uniform precession frequency (not a driving field frequency). The onset of instability is defined by $\left|\Phi_p c_0^2(0)\right| \geq \eta_{\mathbf{k}}$. Taking $m_{x_0}(0) = 0$, from Eqs.(18b), (18c) and (27) we can find $c_0(0)$ and rewrite this criterion as

$$\theta \geq (u_0 + v_0)\sqrt{2\eta_{\mathbf{k}}/|\Phi_p|}, \qquad (55)$$

where $\theta \equiv \tan^{-1}(m_{y_0}/m_{z_0})$ is the initial deviation angle from the equilibrium direction z_0. Here the parameters u_0, v_0 and Φ_p can be directly calculated using Eqs. (27), (50). The damping $\eta_{\mathbf{k}}$ can be estimated microscopically [7], [8].

Fig.2 shows two different evolutions in magnetic system calculated by Eqs.(51a), (51b). Spin-wave pair excitation in Fig.2a is not sufficiently strong during the switching process to affect the uniform magnetization dynamics. The averaged $\langle \mathbf{m}(\mathbf{r}) \rangle = \mathbf{m}_0$, which gives a measure of non-uniform magnetization motions in the system is relatively small $(1 - |\mathbf{m}_0| \lesssim 0.05)$. Fig.2b demonstrates a strong excitation of spin-wave instability by uniform precession and substantial increase of the magnetization reversal rate. In this case $|\mathbf{m}_0|$ reaches $\simeq 0.6$. For stronger coupling $(|\Phi_p|/\eta_{\mathbf{k}} > 21)$ we observed beating between the uniform precession and the spin-wave pair. In this case it is necessary to include into consideration the excitation of another resonance spin-wave pairs.

IV. DISCUSSION

We have considered two principally different scenarios of magnetization reversal. In the first one the coherent rotation of all spins is the principal motion in the system. The magnetization dynamics is defined by Eq. (8), which is reduced to Bloch-Bloembergen form in the case of small magnetization motions.

In the second scenario we have considered an ultra-thing ferromagnetic film with large dimensions in the plane. In this case the role of plane boundaries is negligible and the most convenient technique to describe the non-uniform spin motions is their spin-wave representation in the $\mathbf{k}$-space. Taking into account linear spin-wave modes and their scattering, we have constructed a nonlinear self-consistent theory of magnetization reversal as a decay of uniform magnetization precession and nonlinear excitation of spin-wave pairs. This theory includes an effective energy (41) and dynamic equations (45). The most important spin-wave modes are defined by the resonance condition $2\widetilde{\omega}_0 = \widetilde{\omega}_{\mathbf{k}} + \widetilde{\omega}_{-\mathbf{k}}$ (similar to Suhl's second order instability). The excitation of all spin waves out of this resonance is small and therefore they can be considered as a part of a thermal bath. In the simple example we have demonstrated that strongly excited spin-wave instability can increase substantially the magnetization switching rate.

Note that both scenarios are consistent which each other: in the case of $|\mathbf{m}_0| \to 1$ Eqs.(45) can be reduced to the case of a nonlinear oscillator equation considered in scenario 1. We also emphasize that the relaxation rates of uniform precession and spin waves in both scenarios can be estimated from microscopic physics.

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Figure Captions

Fig.1 Equilibrium coordinates in the film plane.

Fig.2 Time evolution of relative absolute amplitudes: 1- the uniform precession $|c_0(t)|$ without spin-wave excitation, 2 - the uniform precession $|c_0(t)|$ with spin-wave excitation, 3 - excited spin waves $|c_k(t)|$. Curve # 4 describes $|\mathbf{m}_0(t)|$. a) $|\Phi_p|/\eta_{\mathbf{k}} = 9$, b) $|\Phi_p|/\eta_{\mathbf{k}} = 21$. The experimental conditions correspond to Fig.2 in Ref. [10].

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